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ABSTRACT

Presented is an overview of the 1979 Illinois Inventory of Educational Progress (IIEP) for eleventh-grade mathematics. The IIEP is a systematic effort by the Illinois State Board of Education to collect information on the educational achievement of Illinois students in certain areas and to make that information available to educational decision-makers. The IIEP employs an objective-referenced approach, with desired student performance expressed in terms of objectives. Students to be tested are selected in a two-stage random sampling method. Since the IIEP is geared toward determining how groups of Illinois students perform on given tasks, no individual student, teacher, school, or district is identified. As part of the study, teachers of participating students were asked to estimate the percentage of students who would obtain correct answers to individual test items. Of the six measured objectives, it was found that teacher estimates were higher than student scores for only two areas. (MP)

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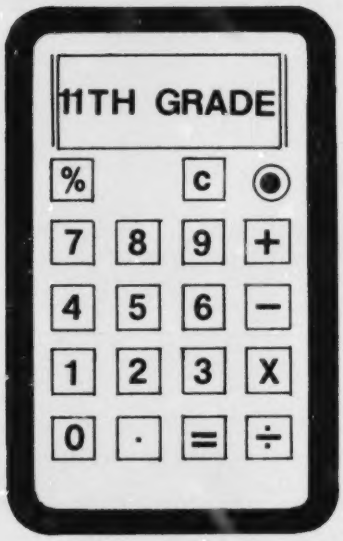

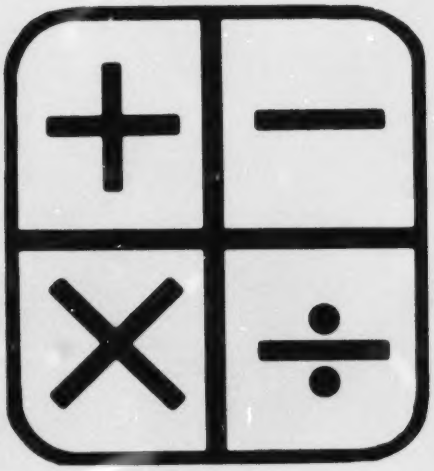

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FOREWORD

What follows is designed to provide an overview of the 1979 Illinois Inventory of Educational Progress (IIEP) in eleventh grade mathematics. The test has been administered by the Illinois State Board of Education since 1976; however, this analytical report is in a new and more usable format.

Development of the IIEP is discussed, and results and analyses of the test administered to eleventh grade students are presented. Results and analyses of fourth and eighth grade tests can be found in separate reports. It is hoped that the information contained here will enhance instruction in Illinois schools.

While many state staff members contributed to the preparation of this report, I would like to especially acknowledge the efforts of Dr. Mervin M. Brennan as the main writer. Any questions concerning this report may be addressed to Dr. Brennan or Dr. Thomas Kerins, Manager of the Program Evaluation and Assessment Section of the Department of Planning, Research and Evaluation of the Illinois State Board of Education.

Donald G. Gill
State Superintendent of Education

PREFACE

Purpose

The Illinois Inventory of Educational Progress (IIEP) is a systematic effort by the Illinois State Board of Education to collect information on the educational achievement of Illinois students in certain areas and to make that information available to educational decision makers.

The three goals of the IIEP are:

1) to make available relevant, reliable, and valid data on the educational attainments of Illinois students;

2) to identify any trends (growth, stability, or decline) in educational attainments which occur over time; and

3) to publish results of the research conducted in connection with the IIEP.

Student Selection

A random sample with two sampling stages is used to select those students attending Illinois public schools who will participate.

First, schools throughout the state are chosen randomly. A sample of fourth, eighth, and eleventh graders is then randomly selected from lists of eligible students submitted by schools for participation. These grade levels are selected to correspond roughly with the end of the primary, elementary, and secondary levels of education.

Since the IIEP is geared toward determining how groups of Illinois students perform on given tasks, no individual student, teacher, school, or district is identified in any reports of the results.

Type of Test

The IIEP employs an objective-referenced approach. An objective-referenced assessment instrument assesses student performance. Desired student performance is expressed in terms of objectives. An objective is a statement of desired student performance, for example: "Fourth grade students should be able to recognize geometric shapes such as circles, etc." Student performance is measured by test items designed to determine whether or not certain groups of students are able to do what the objectives state they should be able to do.

Subject Areas

The IIEP has been in existence since 1976. A number of subject areas have been assessed, for example, reading, mathematics, science, citizenship, energy and nutrition, as well as student attitudes about themselves and education in general.

Base line data is collected during the first year that any subject area is assessed. For each succeeding year that a subject area is reassessed, comparisons can be made concerning student performance on specific objectives, and any growth or decline in achievement can be noted.

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CHAPTER 1

Illinois Inventory of Educational Progress - Mathematics

Development of the 1979 Mathematics-IIEP

In the spring of 1978, a panel of six mathematics educators with elementary, junior high, high school and college teaching and administrative experience was convened to assist State Board staff in formulating the 1979 mathematics IIEP (a roster of panel members appears in Appendix D). Charged with redrafting the objectives which had been developed for the 1976 IIEP, the group met over a two-month period. The results of their work are discussed later in this report.

Additionally, results of a teacher survey that was administered with the previous year's IIEP (1978) were used in developing the 1979 mathematics IIEP. Produced by State Board staff, the survey sought to: (1) validate the test; (2) supply an additional perspective on the results; and (3) provide a standard of performance, based upon teacher estimates, with which student results could be compared.

Toward that end, one mathematics teacher from each school which participated in the IIEP was asked to do three things for each test item. Teachers were asked to determine (1) whether students had been exposed to the material and (2) whether the item was of an appropriate level of difficulty. Teachers were also asked (3) to estimate the percentage of students that could be expected to answer each item correctly. A sample of the teacher survey is contained in Appendix C. Results of the teacher survey are discussed in Chapter 2.

The Test

The test was a domain and objective-referenced test, which means simply that the items tested the general domain of mathematics and that items are derived from or keyed to a set of curricular objectives.

Mathematics objectives for the 1979 IIEP were developed by the aforementioned panel of educators. The following mathematics topics and abilities reflect those objectives. A list of topics precedes a summary description of abilities. Some of the topics are self-explanatory; a brief definition is provided for those which are less common. The abilities are a bit more detailed; essentially, they are the skills required for success in mathematics. Each mathematics objective describes a particular ability with reference to a specific topic.

Mathematics Topics

I. NUMERATION CONCEPTS. This topic refers to the concepts of numeration and place value, and the processes of naming numerals, approximating numbers, and rounding off numbers.

II. PROPERTIES OF NUMBERS AND OPERATIONS. This topic also includes characteristics of numbers and operations and comparisons among numbers.

III. NUMBERS.

A. WHOLE NUMBERS. Whole numbers are the numbers used by children to count. Whole numbers include 0, 1, 2, 3, etc.

B. FRACTIONS.

C. DECIMALS.

D. PERCENT.

E. INTEGERS. Integers are positive and negative whole numbers and zero as distinguished from fractions. The numbers -3, -2, -1, 0, +1, +2, +3, etc., are integers.

F. RATIONALS. Rationals is an all-inclusive term for topics A through E, both positive and negative. Examples are +2, +1/2, +.50, +50%, -2, -1/2, -.50, and -50%.

G. REALS. Reals is an all-inclusive term for topics A through F and numbers such as π, $\sqrt{2}$, etc.

IV. MEASUREMENT.

V. ALGEBRA.

VI. GEOMETRY.

VII. PROBABILITY AND STATISTICS.

VIII. PERSONAL AND CONSUMER MATHEMATICS.

Mathematics Abilities

1. Ability to recall and recognize facts, definitions, and symbols quickly. Perception is the primary mental act used.

2. Ability to perform computations, procedures, and complex counting where the operations are indicated.

3. Ability to understand concepts, facts, and processes. The mental operations of analysis and synthesis are used to make comparisons and evaluative judgments.

4. Ability to solve complex word problems. Several of the following operations must be involved: interpretation of the question, identification of the relevant data from the given information, decisions about which operations need to be performed on the data, correct performance on the operations, and interpretations of the results.

Each mathematics item tested a student ability with respect to one of the mathematics topics. The matrix of mathematics topics and abilities (Table 1) shows the conceptual model of the IIEP mathematics tests. Each cell of the matrix is a specific mathematics objective.

The test contained items on nine topics and four abilities. There were items related to 15 objectives within the topics and abilities. A topic, ability, or objective was considered to be measured if there were three or more items testing it. By that standard the test measured seven topics, four abilities, and six objectives within them. The test is described more fully in subsequent chapters of this report.

Table 1
MATRIX OF MATHEMATICS OBJECTIVES

MATHEMATICS CATEGORIES BY ABILITIES

Mathematics Topics	Mathematics Abilities 1		2		3		4	
	Ability to recall and recognize facts, definitions, and symbols quickly		Ability to perform computations, procedures, and complex counting where the operations are indicated		Ability to understand concepts, facts, and processes		Ability to solve complex word problems	
I. NUMERATION CONCEPTS	1	*4	2		3	8	4	
II. PROPERTIES OF NUMBERS AND OPERATIONS	5	4, 8	6	8	7	4	8	8
III. NUMBERS								
A. WHOLE NUMBERS	9	4	10	4, 8, 11	11	8	12	4
B. FRACTIONS	13	8	14	11	15	11	16	
C. DECIMALS	17		18	8	19		20	
D. PERCENT	21		22	8	23		24	
E. INTEGERS	25		26	4, 8	27		28	8, 11
F. RATIONALS	29	8	30	11	31	11	32	8, 11
G. REALS	33		34		35		36	
IV. MEASUREMENT	37	4, 8 11	38	8, 11	39	8	40	4, 8 11
V. ALGEBRA	41		42	8, 11	43		44	
VI. GEOMETRY	45	8	46		47	8, 11	48	11
VII. PROBABILITY AND STATISTICS	49		50	11	51		52	
VIII. PERSONAL AND CONSUMER MATHEMATICS	53	4	54	4, 8	55	4	56	4, 8, 11

* The numerals (4, 8, 11) indicate the grade level(s) at which these items were tested in the 1979 IIEP.

Chapter 2

ITEM RESULTS

As mentioned in Chapter 1, teachers of participating students were asked to estimate the percentage of students who would obtain the correct answers to the items. The hypothesis was that the teacher estimates would be higher than the student scores. Chapter 4 shows the statistical results.

It was anticipated that there would be some discrepancies between teacher estimates and student scores which could not be submitted to statistical tests or would not reach significance levels, but would lend themselves to suggestions for future research. After statistical analysis of the data, experienced Illinois mathematics educators were asked to comment on the results.

The following descriptions were used for discrepancies between teacher estimates and student scores:

- approximating for discrepancies of ten or less percentage points,
- higher than/lower than for discrepancies of 11 to 20 points, and
- considerably higher than/lower than for discrepancies of more than 20 points.

These discrepancy guidelines were established because consultants suggested the use of consistent standards. Ten percentage points was used since standard deviations for previously calculated data were usually near .10.

The panel of mathematics educators was asked to analyze and interpret the test results using the test data and the teacher survey data. They reflected upon the data for each curricular topic and each objective within the topics. This chapter gives the data and the panel's comments. Correct answers are underlined. Teacher estimates are abbreviated as teach. est., student scores are abbreviated as stu. score.

The comments are solely those of the experts and are not to be taken as the official position of the State Superintendent of Education or the Illinois State Board of Education.

Topic I: Numeration Concepts and
Topic II: Properties of Numbers and Operations

The mathematics panel decided during the test development phase that Topics I and II (objectives 1-8) were of lower priority than other topics for an eleventh grade test. Neither topic was tested in the 1979 IIEP.

Topic III: Numbers

Fifteen items, more than one third of the test, measured this topic. There were four items on whole numbers, seven on fractions, three on rationals, and one on integers.

Whole Numbers All four items on this topic tested computation. Table 2 shows the results.

Table 2

Objective 10: Computation with Whole Numbers

Item 25. $3(2 + 7) =$

a. 6
b. 12 — teach. est.: 81%
c. 13 — stu. score: 80%
d. 23
e. 27

Item 34. What is the SMALLEST number that can be divided by 3, 6, and 9 without a remainder?

a. 9
b. 12 — teach. est.: 78%
c. 18 — stu. score: 77%
d. 36

Item 24.
$$\begin{array}{r} 2037 \\ \times 82 \end{array}$$

a. 167,770
b. 194,334 — teach. est.: 86%
c. 230,034 — stu. score: 73%
d. 167,034

Item 36. Which of the following is equal to 2^3?

a. 5
b. 6 — teach. est.: 67%
c. 8 — stu. score: 72%
d. 9
e. 16

Panel comments: The student average score for objective 10 (76% correct) approximated the teacher estimate (78% correct). Students scored above 70% on all four items.

Fractions

Six items measured student ability to compute with fractions. One item tested student ability to understand fractions. Table 3 shows the results.

Table 3

Objective 14: Computation with Fractions

Item 33. 1/2 x 1/4 =

a. 1/6
b. 1/8 — teach. est.: 83%
c. 2/6 — stu. score: 75%
d. 2/8

Item 29. 2 2/3 =

a. 2/5
b. 4/6 — teach. est.: 83%
c. 4/5 — stu. score: 74%
d. 4/3
e. 8/3

Item 28. Which of the following pairs of fractions are equivalent?

a. 5/8 and 2/3
b. 5/6 and 2/3 — teach. est.: 74%
c. 4/5 and 14/15 — stu. score: 72%
d. 3/5 and 9/15
e. 1/2 and 14/24

Item 31. 1/2 + 1/3 =

a. 1/5
b. 2/6 — teach. est.: 74%
c. 4/5 — stu. score: 55%
d. 5/6

Item 32. 1 2/5 - 1/2 =

a.	2/3		
b.	9/10	teach. est.:	67%
c.	1 1/10	stu. score:	55%
d.	1 1/7		
e.	1 1/3		

Item 30. Which fraction is the GREATEST?

a.	2/3		
b.	3/4	teach. est.:	69%
c.	4/5	stu. score:	41%
d.	5/8		

Objective 15: Understanding of Fractions

Item 40. There are 13 boys and 15 girls in a group. What fractional part of the group is boys?

a.	13/15		
b.	13/26	teach. est.:	70%
c.	15/28	stu. score:	52%
d.	13/28		

Panel comments: Student scores were significantly lower than the teacher estimates on computations with fractions ($p < .05$), and lower than the teacher estimate for understanding of fractions (See Table 13 for average teacher estimate and student scores). The student score was higher for the fraction multiplication item than for the fraction addition and subtraction items. Scores were also higher for the items requiring identification of equivalent fractions and conversion of a mixed number to a fraction (solution processes apparently known by students), than for conversions of four fractions to a common scale for size comparisons (a solution process which may be known by fewer students).

Integers

The test contained one item on integers. Table 4 shows the results.

Table 4

Results for the Integer Item

Objective 28: Problem Solving with Integers

Item 37. The air temperature on the ground is 31 degrees. On top of a nearby mountain, the temperature is -7 degrees. How many degrees difference is there between these two temperatures?

a.	24 degrees		
b.	4 3/7 degrees	teach. est.:	72%
c.	31 degrees	stu. score:	66%
d.	38 degrees		

Panel comments: This is a real-life problem with which most students are probably familiar. The student score approximated the teacher estimate.

Rationals There were three items on rationals. Each item tested a different student ability. Table 5 shows the results.

Table 5

Objective 30: Computation with Rationals

Item 26. 3/5 =

a.	6 percent		
b.	15 percent	teach. est.:	70%
c.	23 percent	stu. score:	53%
d.	30 percent		
e.	60 percent		

Objective 31: Understanding of Rationals

Item 27. What fraction of $1 is 20 cents?

a.	1/20		
b.	1/9	teach. est.:	74%
c.	1/5	stu. score:	70%
d.	1/4		
e.	1/2		

Objective 32: Problem Solving with Rationals

Item 38. Using a scale of 1 yard = 1/2 inch, a drawing of a swimming pool 25 yards long is made. How many inches will the length of the pool take up in the drawing?

a.	12 1/2
b.	20
c.	25
d.	25 1/2

teach. est.: 64%
stu. score: 82%

Panel comments: Student scores were highest for the problem-solving item (item 38), somewhat lower for the item related to understanding of rationals (item 27), and lowest for the computation item (item 26). These results led to several thoughts and suggestions. The items themselves seemed to vary from concrete to abstract. Also, students could have seen them as varying from being real-life and practical to purely academic. It was suggested that considerable research needs to be done to identify what difficulties are inherent in the mathematics of rationals. The perceptions of students as they attempt these types of items should also be investigated.

Topic IV: Measurement

Ten items (25%) of the total test assessed student abilities in regard to mathematical measurement. Four items tested recognition of measurement facts, one item was on computation in measurement, and five items measured problem solving. Table 6 shows the results.

Table 6

Measurement Items and Results

Objective 37: Recognition of Measurement Facts

Item 53. In the United States, we usually buy gasoline by the gallon. In France, where the metric system is used, people buy gasoline by the

a. meter.
b. liter. — teach. est.: 67%
c. quart. — stu. score: 88%
d. gram.

Item 54. In the United States, we usually buy potatoes by the pound. In Germany, where the metric system is used, people buy potatoes by the

a. meter.
b. liter. — teach. est.: 70%
c. pound. — stu. score: 84%
d. kilogram.

Item 55. Which is the closest to the size of one cm^2?

a. A tennis court
b. Your thumbnail — teach. est.: 63%
c. A slice of bread — stu. score: 76%
d. The cover of a record album

Item 62. The perimeter of a square could be measured in

a. square centimeters.
b. liters.
c. degrees Celsius.
d. grams.
e. meters.

teach. est.: 59%
stu. score: 42%

Objective 38: Computation in Measurement

Item 56. About how long is the paper clip above the metric ruler?

a. 30 mm
b. 30 cm
c. 3 m
d. 3 km

teach. est.: 70%
stu. score: 59%

Objective 40: Problem Solving in Measurement

Item 41. What is the volume of a box with a width of 9 cm, a length of 12 cm and a height of 25 cm?

a. 46 cm^3
b. 460 cm^3
c. 270 cm^3
d. 525 cm^3
e. 2700 cm^3

teach. est.: 66%
stu. score: 54%

Item 60. At four o'clock, the size of the angle between the minute hand and the hour hand of a clock is

a. 45 degrees.
b. 60 degrees.
c. 90 degrees.
d. 120 degrees.
e. 150 degrees.

teach. est.: 51%
stu. score: 51%

Item 50. A car takes 15 minutes to travel ten kilometers. What is the speed of the car?

a. 30 kilometers per hour
b. 40 kilometers per hour
c. 60 kilometers per hour
d. 90 kilometers per hour
e. 150 kilometers per hour

teach. est.: 56%
stu. score: 48%

Item 45. Mr. Johnson wants to buy carpeting for his living room. The room is square and has a perimeter of 56 feet. What is the area of the room in square feet?

a. 144 square feet
b. 169 square feet
c. 182 square feet
d. 196 square feet

teach. est.: 53%
stu. score: 44%

Item 52. A runner ran 3,000 meters in exactly eight minutes. What was the average speed?

a. 3.75 meters per second
b. 6.25 meters per second
c. 16.0 meters per second
d. 37.5 meters per second
e. 62.5 meters per second

teach. est.: 51%
stu. score: 38%

Panel comments: Three of the four items testing recognition of measurement facts (objective 37), were related to metric units (e.g. kilograms as a unit of weight). The student scores for all three items were higher than the teacher estimates. The student average score was 83%, as opposed to an average teacher estimate of 67%. However, the student score for computing millimeters from centimeters (item 56) was lower than the teacher estimate.

The student scores were highest for objective 37 (average score: 73%), lower for item 56 which was related to objective 38 (score: 59%), and lowest for objective 40 (average score: 47%). It was hypothesized that students usually learn basic measurement concepts, but do not solve measurement problems as successfully. A suggestion was made to develop a test composed of many measurement items to identify specific student abilities and diagnose difficulties in measurement.

Topic V: Algebra

The four test items on this topic measured computation in algebra (objective 42). Table 7 shows the results.

Table 7

Objective 42: Computation in Algebra

Item 57. If x is replaced by 3, than the value of $x^2 - 1$ is

a.	8	
b.	11	teach. est.: 69%
c.	5	stu. score: 73%
d.	2	

Item 58. Solve the following equation:

$3x - 3 = 12$

$x =$

a.	15	
b.	5	teach. est.: 67%
c.	3	stu. score: 71%
d.	9	

Item 59. $(4x - 2)(x - 5) =$

a.	$4x + 2 \times x - 5$	
b.	$4x^2 - 18x - 10$	teach. est.: 51%
c.	$4x - 10$	stu. score: 51%
d.	$4x + 2x - 3$	

Item 35. The solution set of the equation $x^2 = 9$ is

a.	{3}	
b.	{-3}	teach. est.: 60%
c.	{x=3}	stu. score: 41%
d.	{-3,3}	

Panel comments: The student scores approximated the teacher estimates for three of the four items, and for objective 42 as a whole (student average score: 59%; teacher estimate: 62%).

The student score for item 35 was lower than the teacher estimate. Several panel members hypothesized that less than 50% of Illinois students take algebra before the end of high school. That hypothesis was supported by two sets of data. Twenty percent of the teachers in the IIEP survey indicated that their students had received little or no exposure to the material tested by objective 42. Further evidence supporting the hypothesis is contained in The Illinois Census of Secondary School Course Offerings (1977).[1] Data from the survey indicate that only 12.46% of high school students and 6.03% of junior high school students took elementary algebra during the 1976-77 school year. A conservative estimate from these figures would be that 50-60% of Illinois students have not taken elementary algebra by the end of high school.

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[1] The Illinois Census of Secondary School Course Offerings. Springfield, Illinois 62777: Illinois State Board of Education, 1977.

Topic VI: Geometry

The test contained one item on the understanding of geometric concepts (objective 47) and two items on problem solving in geometry (objective 48). Table 8 shows the results.

Table 8

Geometry Items and Results

Objective 47: Understanding Geometric Concepts

Item 63. Which is true?

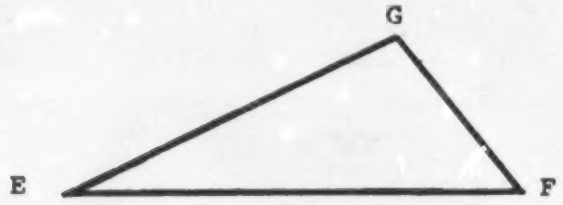
a.	All rectangles are squares.		
b.	All squares are rectangles.	teach. est.:	49%
c.	No squares are rectangles.	stu. score:	38%
d.	No rectangles are squares.		
e.	None of the above.		

Objective 48: Problem Solving in Geometry

Item 61. In a given triangle, the measures of two of the angles are 60 degrees and 70 degrees. The measure of the third angle is

a.	50 degrees.		
b.	60 degrees.	teach. est.:	56%
c.	70 degrees.	stu. score:	60%
d.	90 degrees.		
e.	130 degrees.		

Item 64. If the measure of Angle F is 50° and the measure of Angle G is 105°, what is the measure of Angle E?



a. 25°
b. 35°
c. 55°
d. 180°

teach. est.: 56%
stu. score: 56%

Panel comments: The student scores were higher for the problem solving items than for the item requiring understanding of a geometric concept. The student scores (average score: 58%) approximated the teacher estimates (average estimate: 56%) for the problem solving items, but the student score was lower than the teacher estimate for the item related to conceptual understanding. Several panel members hypothesized that less than 50%, and perhaps less than 33%, of Illinois students take a course in plane geometry before the end of high school. That hypothesis was supported by two sets of data. Twenty-seven percent of the teachers in the IIEP survey reported that their students had received little or no exposure to the material tested by the geometry items. The data of The Illinois Census of Secondary School Course Offerings (1977)[2] indicate that only 7.85% of Illinois High School students and 0.39% of Junior High students were enrolled in a plane geometry course during the 1976-77 school year. A conservative estimate would be that 60-70% of Illinois students do not take plane geometry before the end of high school.

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[2] The Illinois Census of Secondary School Course Offerings. Springfield, Illinois 62777: Illinois State Board of Education, 1977.

Topic VII: Probability and Statistics

The test contained one item on this topic. Students were required to compute simple statistics presented in a table. Table 9 shows the results.

Table 9

Objective 50: Computation with Statistics

Number of Telephones in Operation in Various World Areas in 1968 (0 = 8 million telephones)

Area	Number of Telephones
North America	0000000000000000
Europe	0000000000
Asia	0000

Item 46. According to the chart, in 1968 the number of telephones in operation in North America was how many times the number of telephones in operation in Asia?

a. 3 times as many
b. 2 times as many
c. 5 times as many
d. 4 times as many

teach. est.: 65%
stu. score: 70%

Panel comments: The student score approximated the teacher estimate. Thirty-three percent of the teachers reported in the IIEP survey that their students had received little or no exposure to the material tested by this item.

Topic VIII: Personal and Consumer Mathematics

Eight items tested this topic. All of them measured problem solving related to personal and consumer situations (objective 56). Table 10 shows the item results.

Table 10

Objective 56: Problem Solving in Personal and Consumer Mathematics

Item 47. John's parents bought a refrigerator for $375. If they pay $20 per month for two years, how much more than $375 will the refrigerator cost them?

a.	$ 95	
b.	$105	teach. est.: 68%
c.	$200	stu. score: 80%
d.	$375	

Item 43. Television sets are on sale at two stores. One offers a 10 percent discount while the other offers 15 percent. What is the difference in dollars in the sale price at the two stores of a TV set that is regularly priced at $100?

a.	$ 5	
b.	$10	teach. est.: 63%
c.	$15	stu. score: 78%
d.	$25	

Item 48. An automobile can be bought for cash for $2,850 or on credit with a down payment of $400 and $80 a month for three years. How much MORE would a person pay by buying on credit rather than by buying the car for cash?

a.	$400	
b.	$430	teach. est.: 63%
c.	$450	stu. score: 74%
d.	$470	

Item 44: In a school election with three candidates, Joe received 120 votes, Mary received 50 votes, and George received 30 votes. What percentage of the total number of votes did Joe receive?

a. 40%
b. 50%
c. 60%
d. 70%

teach. est.: 56%
stu. score: 60%

Item 49. Cloth is sold by the square meter. If six square meters of cloth cost $4.80, the cost of 16 square meters will be

a. $12.80
b. $14.40
c. $28.80
d. $52.80
e. $128

teach. est.: 53%
stu. score: 69%

Item 42. Tom bought a bicycle last year for $70. This year the same model is selling for 10% more. What is the price of the bicycle this year?

a. $77
b. $80
c. $82
d. $87

teach. est.: 65%
stu. score: 66%

Item 39. A door-to-door salesperson receives 20 percent of the retail value of his/her sales as commission. What must his/her total retail sales be if he/she is to earn a commission of $60?

a. $120
b. $200
c. $250
d. $300

teach. est.: 53%
stu. score: 57%

Item 51. The price of an article was $100. The price was first raised by 10% and was then reduced by 10% of the new price. What is the price of the article now?

a. $90
b. $99
c. $100
d. $101
e. $110

teach. est.: 50%
stu. score: 31%

Panel comments: The average student score (66%) approximated the average teacher estimate (59%). Panelists noted that the range of scores was quite large, - from 80% for item 47 to 31% correct for item 51. It was suggested that research needs to be done regarding how student success is affected when items are highly relevant to their everyday lives vs. when items are less relevant. One hypothesis could be that the more relevant to everyday life a problem is, the more successful students will be in solving it.

Another suggestion was that research is necessary regarding student abilities to work with percentages. Scores on percent items were 78% (item 43), 69% (item 44), 66% (item 42), 57% (item 39) and 31% (item 51). Why was the range of scores so large? Do students understand the concept of percent? Do they know the mechanics for working with percentages? What instructional approaches are most helpful to engender developed abilities in percent?

Chapter 3

Discussion of the Results

The mathematics panel was asked to reflect upon and discuss the results of the 1979 test. Their comments were based primarily on the following sources: 1) the performance of 4th, 8th and 11th grade Illinois students on the 1976, 1978, and 1979 IIEP mathematics tests, 2) the IIEP teacher surveys, 3) The Illinois Census of Secondary School Course Offerings (1977), and 4) the findings of mathematical and educational research relevant to the IIEP.

Two major competing hypotheses were explored by the 1979 eleventh grade IIEP: 1) that there would be three ability factors or, 2) that there would be six topic factors. Additionally, it was hypothesized 3) that the teacher estimates would be higher than the student scores for the items loading on whatever factor(s) emerged and, 4) that the teacher estimates would be higher than the student scores for the items that tested the six objectives which were measured.

Neither of the first two hypotheses was supported. Factor analysis revealed one factor, which could probably be called a general mathematics factor. Comparison with the 4th and 8th grade IIEP factor analysis results led to the hypothesis that the emergent factor might be labeled "previously learned mathematics," and was more likely to be an ability factor than a topic factor. It was suggested that future IIEP tests try to identify constructs which may relate to the learning of mathematics.

Hypothesis 3 was not supported. Teacher estimates were not significantly higher than student scores for Factor I.

Hypothesis 4 was supported for objective 14 (computation with fractions) and objective 40 (problem solving in measurement), but not for the other four objectives which were measured. Teacher estimates for objectives 10 and 14 were significantly higher than student scores. Correlations between the two were also significant (r .81 for objective 14 and r .66 for objective 40).

Mathematics scores on the IIEP were higher in 1979 than in 1976 as a whole, (see the 1979 IIEP Annual Report for the trend data). Trends were analyzed for mathematics in general, and that was an excellent beginning. A recommendation for the future was that hypotheses be tested regarding trends for specific mathematical abilities, topics, and/or objectives. The base line data is contained in the 1976, 1978, and 1979 IIEP tests. Have the curricular emphases of the past four or more years brought about improved student performance in specific abilities such as computation or specific topics such as geometry? In the future the IIEP should focus on narrower areas within mathematics and test hypotheses about where improvement is occurring. Such an approach could indicate where students are improving and where changing curricular emphases are effective. Sound tests in the specific areas could be developed. Such a "test bank" would, in itself, be a great benefit. The knowledge gained about students from those tests would be an even greater benefit.

Chapter 4

Factor Analysis Results

The IIEP was first administered in 1976. Results from the test gave base line data regarding mathematics achievement. In 1978, the objectives were revised in terms that were more easily understood and more amenable to research on learning processes as they occur in students. Results were subjected to factor analysis, a statistical procedure which helps identify student abilities and strategies used in learning.

Factor analysis is a highly technical mathematical and statistical procedure which cannot be fully explained here. However, an intuitive understanding of factors and their derivation is possible. Fred Kerlinger, in his book Foundations of Behavioral Research (1973) wrote:

> Factor analysis is a method for determining the number and nature of the underlying variables among large numbers of measures.
>
> Generally speaking, if two tests measure the same thing, the scores obtained from them can be added together. If, on the other hand, the two tests do not measure the same thing, their scores cannot be added together. Factor analysis tells us, in effect, what tests or measures can be added and studied together rather than separately. It thus limits the variables with which the scientist must cope. It also (hopefully) helps the scientist to locate and identify unities or fundamental properties underlying tests and measures.
>
> A factor is a construct, a hypothetical entity that is assumed to underlie tests and test performance. A number of factors have been found to underlie intelligence, for example: verbal ability, numerical ability, abstract reasoning, spatial reasoning, and memory.

A HYPOTHETICAL EXAMPLE

Suppose we administer six tests to a large number of seventh grade pupils. We suspect that the six tests are not measuring six, but some smaller number of variables. The tests are: vocabulary, reading, synonyms, numbers, arithmetic (standardized tests), and arithmetic (teacher-made tests). The names of these tests indicate their nature. We label them respectively, V, R, S, N, AS, AT. (The last two tests, though both arithmetic, have different contents and reliabilities. We assume a good reason for including them both in a test battery.) After the tests are administered and scored, coefficients of correlation are computed between each test and every other test. We lay out the r's in a correlation matrix (usually called R matrix). The matrix is given in Table 37.1 (Table 11).

Table 11

TABLE 37.1 R MATRIX: COEFFICIENTS OF CORRELATION AMONG SIX TESTS

		V	*R*	*S*	*N*	*AS*	*AT*
Cluster I	*V*		.72	.63	.09	.09	.00
	R	.72		.57	.15	.16	.09
	S	.63	.57		.14	.15	.09
	N	.09	.15	.14		.57	.63
	AS	.09	.16	.15	.57		.72
	AT	.00	.09	.09	.63	.72	

Cluster II

...How many underlying variables or factors are there?...The factors are presumed to be underlying unities between the test performances. They are reflected in the correlation coefficients. If two or more tests are substantially correlated, then the tests share variance. They have common factor variance. They are measuring something in common.

...There are two factors. This is indicated by the clusters of r's circled and labeled I and II in Table 37.1. Note that V correlates with R,.72; V with S,.63; and R with S,.57. V, R, and S appear to be measuring something in common. It is important to note, however, that the tests in Cluster I, though themselves intercorrelated, are not to any great extent correlated with the tests in Cluster II. Likewise, N, AS, and AT, though themselves intercorrelated, are not substantially correlated with the tests V, R, and S. What is measured in common by the tests in Cluster I is evidently not the same as what is measured in common by the tests in Cluster II. There appear to be two clusters or factors in the matrix. (pp. 659-661).[3]

For further discussion of factor analysis, see Kerlinger (1973) pp. 659-692 and cited references.

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[3] Fred N. Kerlinger. Foundations of Behavioral Research (2nd Edition). New York: Holt, Rinehart, and Winston, Inc., 1972.

Results for Factors and Hypotheses

Hypothesis 1 stated that four ability factors would be indicated. The hypothesized factors were. 1) recognition of mathematical facts, 2) computational skills, 3) an understanding of concepts, and 4) problem solving ability. The data showed one factor.

The fourth and eighth grade data had indicated two factors. See Fourth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress[3] and Eighth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress.[5] In each case one factor was comprised of items that required students to deal successfully with "previously learned material," and the other factor was comprised of items that required students to do "problem solving which required original thinking." The eleventh grade factor was comprised of items that required students to deal successfully with "previously learned material."

Hypothesis 2 stated that there would be seven topic factors, one factor for each topic which was measured. The hypothesis was not supported. No topic factor was indicated.

Hypothesis 3 stated that the teacher estimates would be higher than the student scores for the factors. The hypothesis was not supported. Table 12 shows the results.

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[4]Fourth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress. Springfield, Illinois 62777: Illinois State Board of Education, 1981.

[5]Eighth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress. Springfield, Illinois 62777: Illinois State Board of Education, 1981.

Table 12

Factor I: Learned Material

Objective	Item	Teacher Estimate	Student Score
10	25	81%	80%
	34	78%	77%
	24	86%	73%
	36	67%	72%
14	33	83%	75%
	29	83%	74%
	28	74%	72%
	31	74%	55%
	30	69%	41%
15	40	70%	52%
28	37	72%	66%
30	26	70%	53%
31	27	74%	70%
32	38	64%	82%
37	53	67%	88%
	54	70%	84%
	55	63%	76%
	62	59%	42%
38	56	70%	59%
40	41	66%	54%
	60	51%	51%
	50	56%	48%
	45	53%	44%
	52	51%	38%
42	57	69%	73%
	58	67%	71%
	59	51%	51%
	35	60%	41%
50	46	65%	70%
56	47	68%	80%
	43	63%	78%
	48	63%	74%
	44	56%	69%
	49	53%	69%
	42	65%	66%
	39	53%	57%
	51	50%	31%
Mean		.66	.63
Standard Deviation		.10	.15
N		38	38

$t = .843+$ $df = 74$ $r^2 = .004$

+N.S.

Results for Specific Objectives

Hypothesis 4 stated that the teacher estimates would be higher than the student scores for the items of the six measured objectives. The hypothesis was confirmed for objective 14 (computation with fractions), and objective 40 (problem solving in measurement), but not for the other four objectives. The degree of difference between the teacher estimate and student performance is illustrated by the size of the r^2. Table 13 shows the results.

Table 13

T-test Results for Specific Objectives

Objective	Teacher Estimates		Student Scores					
	Mean	S.D.	Mean	S.D.	N	T-test results	df	r^2
10	.78	.08	.76	.04	4	.57+	6	--
14	.75	.07	.62	.14	6	2.07*	10	.21
37	.65	.05	.73	.21	4	-.70+	6	--
40	.55	.06	.47	.06	5	2.03*	8	.31
42	.62	.08	.59	.16	4	.34+	6	--
56	.59	.07	.66	.16	8	-1.16+	14	.02

*significant at $p < .05$ level
+ N.S.

Additional Analyses

In order to identify hypotheses for future research, post hoc analyses of the data were done. Correlations were computed between teacher estimates and student scores to explore the relationship between the two. This was done for the factor as a whole and for each measured objective.

All resulting correlations, except for objective 10 (computations with whole numbers), were significant. The degree to which student performance was accounted for by the teacher estimates can be seen from the size of the r^2. Table 14 shows the results.

Table 14

Teacher Estimates Correlated with
Student Scores by Factor and Objective
for the 1979 Eleventh Grade IIEP

	r	df	r^2
FACTOR I	.54***	74	.29
Objective 10	.36+	6	.13
Objective 14	.81**	10	.66
Objective 37	.88***	6	.77
Objective 40	.66*	8	.44
Objective 42	.74*	6	.55
Objective 56	.75***	14	.55

* significant $p < .05$

** significant $p < .01$

*** significant $p < .001$

\+ N.S.

T-tests were computed to test for significant differences among objectives. Three significant differences were found. Scores for problem-solving in measurement (objective 40) were significantly lower than scores for computation of whole numbers (objective 10), recognition of measurement facts (objective 37), and problem solving in consumer mathematics (objective 56). The degree of difference can be seen by the size of the r^2. The difference between student performance on objectives 10 and 40 is particularly strong ($r = .88$). Table 15 shows the results.

Table 15

T-test Results for Objectives Measured by the 1979 Eleventh Grade IIEP

Obj.	Mean	S.D.	N	Obj.	Mean	S.D.	N	t	df	r^2
10	.76	.04	4	14	.62	.14	6	1.88+	8+	.21
10	.76	.04	4	37	.73	.21	4	.28+	6+	--
10	.76	.04	4	40	.47	.06	5	8.07**	7**	.88
10	.76	.04	4	42	.59	.16	4	2.06+	6+	.25
10	.76	.04	4	56	.66	.16	8	.17+	10+	--
37	.73	.21	4	14	.62	.14	6	.98+	8+	--
37	.73	.21	4	40	.47	.06	5	2.63+	7*	.40
37	.73	.21	4	42	.59	.16	4	1.04+	6+	.01
37	.73	.21	4	56	.66	.16	8	.61+	10	--
56	.66	.16	8	14	.62	.14	6	.50+	12+	--
56	.66	.16	8	40	.47	.06	5	2.55*	11*	.30
56	.66	.16	8	42	.59	.16	4	.74+	10+	--
14	.62	.141	6	40	.47	.06	5	2.24	9+	.27
14	.62	.141	6	42	.59	.16	4	.32	8+	--
42	.59	.161	4	40	.47	.06	5	1.60+	7+	.15

*significant $p < .05$

** significant $p < .001$

+ N.S.

APPENDIX A

INDEX OF MATHEMATICS OBJECTIVES
FOR THE 1979 ELEVENTH GRADE IIEP

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37	54	12
37	55	12
37	62	13
38	56	13
40	41	13
40	45	14
40	50	14
40	52	14
40	60	14
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APPENDIX B

INDEX OF MATHEMATICS ITEMS
FOR THE 1979 ELEVENTH GRADE IIEP

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27	30	10
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APPENDIX C

STATE BOARD OF EDUCATION
ILLINOIS OFFICE OF EDUCATION
Program Evaluation and Assessment Section
100 North First Street
Springfield, Illinois 62777

(1) (2) (3) (4)

11th GRADE MATH ATTENDANCE CENTER TEACHER SURVEY

INSTRUCTIONS: Starting with Column 8, indicate your response by placing a number corresponding to your opinion in the appropriate box. Return the form to your building principal when completed.

ITEM NUMBER (5-7)	WHEN WERE STUDENTS EXPOSED TO THE SUBJECT MATTER? 1 Exposed prior to this grade level 2 Exposed this year 3 Have not been exposed (8)	TO WHAT DEGREE HAVE STUDENTS BEEN EXPOSED TO THE SUBJECT MATTER? 1 Heavily 2 Adequately 3 Minimally (9)	HOW IMPORTANT IS MASTERY OF THIS SKILL? 1 Very Important 2 Important 3 Not Important (10)	DOES THE EXERCISE MEASURE SUBJECT MATTER? 1 Yes 2 No (11)	EXERCISE DIFFICULTY INDEX: 1 Too Easy 2 Appropriate Level of Difficulty 3 Too Difficult (12)	WHAT PERCENTAGE OF STUDENTS WILL ANSWER THIS ITEM CORRECTLY? (13-15)
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ITEM NUMBER (5-7)	WHEN WERE STUDENTS EXPOSED TO THE SUBJECT MATTER? 1 Exposed prior to this grade level 2 Exposed this year 3 Have not been exposed (8)	TO WHAT DEGREE HAVE STUDENTS BEEN EXPOSED TO THE SUBJECT MATTER? 1 Heavily 2 Adequately 3 Minimally (9)	HOW IMPORTANT IS MASTERY OF THIS SKILL? 1 Very Important 2 Important 3 Not Important (10)	DOES THE EXERCISE MEASURE SUBJECT MATTER? 1 Yes 2 No (11)	EXERCISE DIFFICULTY INDEX: 1 Too Easy 2 Appropriate Level of Difficulty 3 Too Difficult (12)	WHAT PERCENTAGE OF STUDENTS WILL ANSWER THIS ITEM CORRECTLY? (13-15)
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APPENDIX D

LIST OF MATHEMATICS PANEL

BUSINESS ADDRESS

Mr. Willie D. Anderson
Carbondale Community High School
Carbondale, Illinois

Mrs. Janet Barnard
Parkside Jr. High School
Normal, Illinois

Mrs. Marie Jernigan
Bureau of Mathematics
Chicago Board of Education
Chicago, Illinois

Mr. Wendell Meeks
Educational Consultant
Program, Planning, and Development Section
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Springfield District 186
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Dr. Aurum I. Weinzweig
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Dr. Margariete Montague Wheeler
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Department of Mathematical Science

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Department of Planning, Research, and Evaluation
Illinois State Board of Education

APPENDIX E

LIST OF PUBLICATIONS DESCRIBING THE RESULTS OF THE 1979 IIEP

1979 Illinois Inventory of Educational Progress Annual Report

Fourth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress

Eighth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress

Eleventh Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress

Energy Results of the Fourth, Eighth, and Eleventh Grade Illinois Inventory of Educational Progress

JAS/1714f